Key

Math 1

**4-1 Geometric Sequences**

Name______________________

Date________

*Learning goals:*

- I can convert a sequence into a recursive or explicit formula.
- I can use a formula to find missing terms in a sequence.
- I can determine the common difference/ratio from a sequence.
- I can identify linear and exponential situations and distinguish between the two.
- I can construct a linear or exponential function from an arithmetic sequence, table of values or verbal description.

RECALL:

**Arithmetic Sequences** Recursive: $\begin{cases} a_1 = \\ a_n = a_{n-1} \pm d \end{cases}$ Explicit: $a_n = a_1 + (n-1)d$

1. Continue the following sequences:

a. 4, 8, 12, 16, 20, 24, 28,… $d=4$

b. -2, 10, 22, 34, 46, 58, 70,… $d=12$

c. 8, -15, -38, -61, -84, -107, -130,… $d=-23$

d. 2.3, 5.7, 9.1, 12.5, 15.9, 19.3, 22.7,… $d=3.4$

2. Write a recursive and explicit rule for the sequence in part d.

Recursive

$\begin{cases} a_1 = 2.3 \\ a_n = a_{n-1} + 3.4 \end{cases}$

Explicit

$a_n = 2.3 + (n-1)(3.4)$

3. In your own words, define an ***arithmetic sequence.***

Add or subtract the same number over and over again to get the next number in the sequence.

Give them a seq. a sequence or two to figure out.

**Geometric Sequences** Recursive: $\begin{cases} a_1 = \\ a_n = a_{n-1} \cdot r \end{cases}$ Explicit: $a_n = a_1 r^{n-1}$

A ***geometric sequence*** is a sequence in which each term after the first is found by multiplying the previous term by a constant. In any geometric sequence, the *constant or common ratio* is found by dividing any term by the previous term. The $n^{\text{th}}$ term $(a_n)$ of a geometric sequence with first term $a_1$ and constant ratio $r$ is given by the formula $a_n = a_1 \cdot r^{n-1}$

Consider the sequence $\{4, 12, 36, 108, ...\}$

The *explicit formula* for this sequence is $a_n = 4 \cdot 3^{n-1}$

The *recursive formula* for this sequence is $\begin{cases} a_1 = 4 \\ a_n = g_{n-1} \cdot 3 \end{cases}$

A. Determine which of the following sequences are geometric. If it is a geometric sequence, find the common ratio.

**Example:** 4, 20, 100, 500 $\rightarrow r = \frac{20}{4} = \frac{100}{20} = \frac{500}{100} = \boxed{5}$

1. 7, 14, 28, 56, . . . $r = \frac{14}{7} = \frac{28}{14} = \frac{56}{28} = \boxed{2}$

2. 2, 4, 6, 8, . . . Arithmetic

3. 3, 9, 27, 54, . . . $r = \frac{9}{3} = \frac{27}{9} \neq \frac{54}{27} \rightarrow$ Not arithmetic or geometric

4. 9, 6, 4, $\frac{8}{3}$ $r = \frac{6}{9} = \frac{4}{6} = \frac{\frac{8}{3}}{4} = \boxed{\frac{2}{3}}$

B. Find the next two terms for each **geometric sequence**:

**Example:** 729, 243, 81, 27, 9 $r = \frac{243}{729} = \frac{1}{3}$

1. 20, 30, 45, 67.5, 101.25 $r = \frac{30}{20} = 1.5$

2. 90, 30, 10, $\frac{10}{3}$, $\frac{10}{9}$ $r = \frac{1}{3}$

3. 2, 6, 18, 54, 162 $r = \frac{6}{2} = 3$

C. Find the first four terms of each **geometric sequence** described below:

**Example:** $a_1 = \frac{3}{2}, r = 2$ $\quad \frac{3}{2}, \underset{\frac{3}{2}\cdot 2}{3}, \underset{3\cdot 2}{6}, \underset{6\cdot 2}{12}$

1. $a_1 = 3, r = -2$

   3, -6, 12, -24

2. $a_1 = 12, r = \frac{1}{2}$

   12, 6, 3, 1.5

3. $a_1 = 27, r = -\frac{1}{3}$

   27, -9, 3, -1

D. Find the $n^{th}$ term of each **geometric sequence** described below:

**Example:** $a_1 = 4, \; r = 5, \; n = 3$ $\quad a_3 = 4 \cdot 5^{3-1} = 4 \cdot 5^2 = 4 \cdot 25 = 100$

$\boxed{a_3 = 100}$

1. $a_1 = 4, \; r = 2, \; n = 3$

   $a_3 = 4 \cdot 2^{3-1} = 4 \cdot 2^2 = 4 \cdot 4 = \boxed{16}$

2. $a_1 = 2, \; r = 2, \; n = 5$

   $a_5 = 2 \cdot 2^{5-1} = 2 \cdot 2^4 = 2 \cdot 16 = \boxed{32}$

3. $a_1 = 32, \; r = -\frac{1}{2}, \; n = 6$

   $a_6 = 32 \cdot (-\frac{1}{2})^{6-1} = 32(-\frac{1}{2})^5 = 32\left(-\frac{1}{32}\right) = \boxed{-1}$

E. Find the missing terms of the following **geometric sequences**:

**Example:** 3, 6, 12, 24, 48

$3 \cdot r^4 = 48 \rightarrow r^4 = 16$

$\sqrt[4]{r^4} = \sqrt[4]{16}$

$\boxed{r = 2}$

1. 243, 81, 27, 9, 3, 1

$243 \cdot r^5 = 1$

$r^5 = \frac{1}{243}$

$r = \sqrt[5]{1/243} = \boxed{\frac{1}{3}}$